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On the Possibility of Harmonic Operation of Cyclotron Wave Parametric Amplifiers

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Cyclotron wave amplifiers at the harmonics of the electron cyclotron frequency are investigated. Since the waves on the beam are electrostatic, harmonics are strongly excited in nonrelativistic beams if they are rotating rather than filamentary. These modes at the harmonics can couple to input cuccia couplers, and pump fields which drive parametric amplification, in very much the same way as they do on filamentary beams at the cyclotron frequency. Harmonic cyclotron wave amplifiers have the possibility of giving rise to a new class of devices at millimeter wave frequencies.

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ON THE POSSIBILITY OF HARMONIC OPERATION OF CYCLOTRON WAVE PARAMETRIC AMPLIFIERS

I. Introduction

Cyclotron Wave Electrostatic Amplifiers (CWESA's) and Cyclotron Wave Parametric Amplifiers (CWPA's) are low noise amplifiers, developed simultaneously in the United States and Russia in the late 50's and early 60's. In the United States, work on these were dropped in favor of solid state electronics. However work continued in Russia and it continues to this day. These amplifiers are the input amplifiers for radar and communication receivers in many Russian systems and many systems which Russia exports. These amplifiers work by actively removing noise from the beam, coupling to an electrostatic wave on the beam, amplifying this wave on the beam in some way, and coupling back out into the electromagnetic system. These amplifiers have many potential advantages, some of which are linearity, low noise, wide and electronically controllable dynamic range, protection of the receiver components beyond the amplifier, and very fast response after a microwave overload. There are two basic types of these amplifiers, parametric amplifiers using a temporally oscillating pump called cyclotrom wave parametric amplifiers, CWPA's; and parametric amplifiers using a spatially varying pump. These are called cyclotron wave electrostatic amplifiers, CWESA's. A companion paper, denoted I, reviews the theory and the properties of these amplifiers.[1]

This paper discusses a possible extension of this technology, an extension to high frequency by using beam modes at harmonics of the cyclotron frequency. Although there are many possible applications, we think mostly in terms of a 94 GHz input amplifier in the receiver of a 94 GHz radar. There is now a large program in the United States at the Naval Research Laboratory to put together such a radar and to investigate its properties. This radar can use either millitrons, or gyroklystrons as the transmitter tube. One possible application for CWPA's or CWESA's at millimeter wavelength would be as input amplifiers and receiver protectors for such a radar. For instance receiver protectors at 94 GHz at high power have not yet been optimally developed. Furthermore, a 94 GHz Doppler radar would require an extremely high pulse repetition rate (PRF) in order to provide adequate Doppler space for clutter suppression. This means the receiver protector would need a very fast recovery time, certainly less than 100ns. Harmonic CWPA's could meet both of these requirements. Furthermore, at 94 GHz, receivers (either low noise amplifiers or mixers) at 94 GHz have noise figures of around 5-6 dB, at least if they are not cryogenically cooled. Harmonic CWPA's 94 GHz have the possibility of developing into input amplifiers with perhaps 1-2 dB noise figure. Thus harmonic CWPA's could give a potentially unique solution to many of the difficulties facing 94 GHz radar receiver technology. Furthermore, they could be developed even more easily at lower millimeter wave frequencies such as 35, 44 and 60 GHz, and these could have potential applications to space based communications and surveillance systems.

The cyclotron wave amplifiers amplify waves at the cyclotron frequency. They achieve low noise temperature in one of two ways. The CWPA's manage to cool the beam entirely in the coupler. The CWESA's rely on propagating the beam into an expanding magnetic field so as to adiabatically cool it. Clearly this latter technique

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becomes more and more difficult for millimeter waves, because even at the harmonics, the required magnetic field is large. For instance a second harmonic device at 94 GHz would require a magnetic field of about 17 kG. To provide significant adiabatic cooling, the magnetic field at the cathode would have to be considerably larger. This does not seem easy, or even possible with existing magnetic materials. Therefore, for millimeter wave systems, we concentrate on CWPA's, which have no need for larger fields at the cathode.

There has been much discussion and experiments regarding cyclotron devices, for instance gyrotron oscillators and amplifiers, at harmonics of the cyclotron frequency.[2-4] Also free particle cyclotron emission can occur at the harmonics as well[5]. However in every case, this harmonic radiation results from the relativistic nature of the particle dynamics. Therefore it is much more pronounced at high energy. Electrostatic interactions, on the other hand, can give rise to harmonics at very low beam energy. This was first pointed out by Bernstein[6], who showed that there are electrostatic modes which propagate perpendicular to the magnetic field at every harmonic of the cyclotron frequency. Furthermore, these modes exist for any plasma temperature other than zero. Harmonic cyclotron emission has been found in many warm plasmas, but at an electron temperature much too low for harmonic generation by single particle cyclotron radiation. For example, Landauer[7] reports observation of about 40 harmonics from low pressure gas discharges; Crawford Kino and Weiss[8] observe about 10 harmonics in microwave transmission across a plasma; and multiple harmonic interactions in ionoshperic top side soundings have been reported[9].

Hirshfield [10-14] and his coworkers have invoked electrostatic interactions at the cyclotron harmonics for years, both to explain experiments, and also to develop low voltage microwave tubes at the cyclotron harmonics. If this could be done, it would be a very major advance, because such tubes, gyrotrons for instance, need high magnetic field, high beam energy, or both to efficiently produce radiation at high frequency. One difficulty however, is that the way in which electrostatic waves on the beam couple to electromagnetic waves in free space or in the appropriate electrodynamic structure, which they must do if they are to produce a useful microwave tube, is really unknown.

It is natural to think that cyclotron wave electrostatic amplifiers, which use electrostatic waves on a beam might also be able to operate at harmonics. Furthermore, the way these modes couple to the external electromagnetic system is also well known, and has been since Cuccia discovered his coupler in 1949[15]. This paper investigates the possibility of cyclotron wave amplifiers at the harmonics, where the coupling back and forth to the external electromagnetic system is also specified. We find that the beam can have two possible equilibria. The first is a filamentary equilibrium, where the beam electrons propagate along the magnetic field in the normal way. The second is a rotating equilibrium where the electrons rotate around the axis, and have an axial velocity also. For a low density beam, this rotation is at the electron cyclotron frequency. We find that a rotating beam has normal modes at all harmonics of the cyclotron frequency, where the filamentary beam only has modes at Ω_c, the cyclotron frequency. Furthermore, the mode

at $2\Omega_c$ on the rotating beam, acts almost exactly like the mode at Ω_c on the filamentary beam as regards both parametric instability and interaction with the Cuccia coupler. Thus if one has a rotating beam, a doubling of the frequency is virtually free. At higher harmonics, the growth rates fall off, but very slowly, and the coupling would be to Cuccia like couplers but with different azimuthal symmetry.

This paper analyzes what the author believes to be the first example of a specific device exploiting electrostatic waves at the cyclotron harmonics, which also analyzes the coupling to an electromagnetic structure. As such, it seems to be of fundamental interest. However it may also have practical applications as input amplifiers for millimeter wave radar and communication systems. Section II discusses waves on filamentary and rotating beams, Section III discusses the coupling of these waves to external Cuccia like couplers, Section IV discussed the parametric instabilities, and Section V discusses how these rotating beams may be produced.

II. Electrostatic Waves on a Tenuous Rotating Beam

We begin by considering electrostatic waves on a rigidly rotating electron beam with very low charge density. As in I, the waves on this beam are determined only by the electron dynamics, the electrostatic fields produced by the perturbed beam motion are assumed to be negligible. Also, we consider only a solid rotating beam with uniform charge density. While this beam is most likely more difficult to produce than an annular rotating beam, it has certain advantages in that if the electron distribution function is Maxwellian, the beam is stable to all electrostatic perturbations[16]. Also it is relatively simple to analyze. Later on, we consider issues regarding solid and annular beams. The equilibrium and electrostatic oscillations of such beams have been worked out now in textbooks[17]. However the theory of these electrostatic oscillations for the case of neglect of self fields is so simple compared to the standard treatment that it is worth briefly sketching out here.

The equation for the electron velocity is

$$\partial \mathbf{v}/\partial t + \mathbf{v}\bullet\nabla\mathbf{v} + \boldsymbol{\Omega}_c\times\mathbf{v} = -e/m\nabla\phi \qquad (1)$$

where ϕ is the electrostatic potential which drives the system or else couples different waves on the beam in the parametric instability. To calculate the equilibrium and waves on the beam in the low density limit, we assume $\phi=0$. If the time dependence vanishes, there are two possible equilibrium solutions for the velocity

$$\mathbf{v} = v_o\mathbf{i}_z \qquad (2a)$$

$$\mathbf{v} = v_o\mathbf{i}_z + r\Omega_c\mathbf{i}_\theta \qquad (2b)$$

The former, (2a) is the normal filamentary beam equilibrium discussed in I. The latter, (2b) is the other possible equilibrium, a rigidly rotating beam where the electrons rotate at the cyclotron frequency. We denote them as filamentary and rotating equilibria. These two equilibria are indicated schematically in Fig.(1).

In equilibrium, the electron density is any function of radius, since there is no radial velocity. However a rigid rotor equilibrium (when self fields are considered) is only consistent with a uniform density up to $r = r_b$, at which point it drops sharply to zero. We treat only this case, but speak briefly later on of annular beams. At non zero electron charge density, the filamentary equilibrium actually has a rotation frequency given by the ExB drift velocity. The rotating equilibrium has a rotation frequency which is reduced from Ω_c by the presence of electrostatic fields of the beam. As the charge density of the beam is increased, these two frequencies join at the Brillouin limit, that is the maximum beam density that can be confined by a specified magnetic field[18]. At this beam density, the two roots for the rotation combine at $\Omega_c/2$. However for the purposes of this work, we consider only the case of vanishingly small electron density.

To account for the two possible solution to the equilibrium, we use the notation for the equilibrium velocity

$$\mathbf{v} = v_o\mathbf{i}_z + r\text{ɥ}\mathbf{i}_\theta \qquad (3)$$

where ɥ is zero for the filamentary beam, or is Ω_c for the rigidly rotating beam. If non zero charge density is considered, ɥ denotes whatever the rotation frequency is. Note that in our sign convention, both Ω_c and ɥ are always defined as being positive.

We now consider perturbations to this equilibrium. Assume the perpendicular velocity is given a perturbation, denoted v_r and v_θ, which varies in space as $\exp i(kz + p\theta - \omega t)$, where as in I our convention is that ω is positive but both k and p can take either sign. Then the perturbed r and θ components of the momentum equation are

$$-i\omega' v_r + \Omega' v_\theta = 0 \qquad (4a)$$

$$\Omega' v_r + i\omega' v_\theta = 0 \qquad (4b)$$

where $\omega' = \omega - p\text{ɥ} - kv_o$, and $\Omega' = \Omega_c - 2\text{ɥ}$. Thus the dispersion relation is simply

$$\omega' = \pm\,\Omega' \qquad (5)$$

For the case of ɥ = 0, we have the dispersion relation for the positive and negative energy cyclotron waves

$$\omega = kv_o \pm \Omega_c \qquad (6)$$

On the other hand, for the case of ɥ = Ω_c, we have

$$\omega = kv_o + (p\pm 1)\,\Omega_c \qquad (7)$$

Note that this dispersion relation includes only the cyclotron waves, and not the synchronous waves discussed in I. These latter waves on the beam produce only displacement, not velocity perturbations. As they play no role in the amplifying or coupling structures of this paper, we do not consider them further here.

Note that the rotating beam, as opposed to the filamentary beam, has normal modes at all harmonics of the electron cyclotron frequency. These are analogous the Bernstein modes in hot electron or ion plasmas. The dispersion relation for electrostatic perturbations of a rigidly rotating beam including self electric fields has been given in for instance Davidson[17]. The calculation is much more complicated and the final dispersion relation cannot be written out in nearly as simple a form. However, as pointed out there, one finds basically the same qualitative result, namely that there are electrostatic modes at each cyclotron harmonic. What is particularly notable is that for electrostatic perturbations, one can excite high harmonics without relativistic effects. For the

interaction of electromagnetic radiation with particles, whether in gyrotrons or in cyclotron radiation, production of harmonics is always a relativistic effect, implying that a very energetic beam is needed for harmonic operation. This is not the case for electrostatic waves, as we have just derived for vanishing self field, and as can be derived with more effort if self fields are included.

Note that for the rotating equilibrium, if p=1, the analog of the fast cyclotron wave at k=0 is at twice the cyclotron frequency. Thus by exciting this wave, it could be a relatively simple matter to couple to, and amplify waves at $2\Omega_c$.

Now let us consider the polarization of these electrostatic waves. This involves the relation between v_r and v_θ. As is apparent from Eq.(4) and the dispersion relation, the polarization is given by

$$v_r = -i[\Omega'/\omega']v_\theta = -\pm[\Omega'/|\Omega'|]\, v_\theta \qquad (8)$$

where the dispersion relation is $\omega' = \pm|\Omega'|$. If u=0, the filamentary equilibrium, $v_r = -\pm v_\theta$, where the upper sign corresponds to the fast cyclotron wave. On the other hand for the rotating equilibrium, $u=\Omega_c$, and Ω' is negative. Thus in this case, $v_r = \pm v_\theta$, so the polarization relations are reversed as compared to the filamentary equilibrium.

We now consider the energy and power relations in the beam. These waves on the beam are assumed to be set up by an external potential. Since this external potential is a vacuum field, it satisfies Laplace's equation

$$r d/dr\, r d\phi/dr - p^2\phi = 0 \qquad (9)$$

where we have assumed $kr_b \ll 1$. The spatial and temporal dependence of the driving potential is then

$$\phi = \phi(t) r^p \exp i(kz + p\theta - \omega t) \qquad (10)$$

The energy input into the fluid per unit length is then

$$W_f = \int^t dt' \int d^2r\, \mathbf{J}\bullet\mathbf{E}^* + c.c. = -\int^t dt' \int d^2r\{\nabla\bullet[\mathbf{J}\phi^*] - \phi^*\, \nabla\bullet\mathbf{J}\} \qquad (11)$$

where we have used the fact that the electric field is negative the gradient of a potential. The divergence term in Eq. (11) vanishes because the beam is limited in r, and also because of the periodicity in k which limits the volume integral to a two dimensional area integral. Then we find that

$$W_f = -\int^t dt' d^2r\, \phi^* \partial\rho/\partial t \qquad +c.c. \qquad (12)$$

where ρ is the perturbed beam charge density. If the potential has the functional form given by Eq.(10), it is not difficult to show from the momentum equation, Eq.(4) that the

fluid motion is incompressible. Since the unperturbed density is constant as a function of radius, there is no perturbed density except at the edge of the beam. Using the fact that the equation for the surface charge density σ is

$$d\sigma/dt = n_o v_r \qquad (13)$$

where d/dt is the derivative following the unperturbed fluid element (ie $d/dt = -i\omega'$). Thus we find

$$\sigma = n_o e v_r / i\omega' \qquad \text{or} \qquad \rho = n_o e \delta(r-r_b) v_r / i\omega' \qquad (14)$$

Now we wish to insert ρ from Eq.(14) into the expression fluid energy, so

$$W_f = -\int^t dt' d^2r\, \phi^*(\omega/\omega')\, n_o e \delta(r-r_b) v_r \qquad (15)$$

Since we would like to express the energy in terms of the velocity perturbation, we express ϕ^* in terms of v_r. Using the equation for v_r, where the time dependence now implies the time dependence after the exp-$i\omega t$ is factored out, we find

$$\phi^* = mr(dv_r^*/dt)/e\,|p| \qquad (16)$$

so that on doing the time integral in Eq.(15) we find

$$W_f = 2\pi r_b^2(\omega/\omega'\,|p|)\, nm\, |v_r^2| \qquad (17)$$

Note that the sign of the wave energy is determined by the sign of ω'. For $\omega'>0$, the energy put into the fluid motion by the external driving potential is positive, and the wave has positive energy. For both the rotating and filamentary beam, this is the fast cyclotron wave which has positive energy, and the slow cyclotron wave which has negative energy.

III. Coupling to the Beam Modes

In this section we will show that the coupling of the p=1 k=0 beam mode to a Cuccia coupler[15] is the same whether the beam is filamentary or rotating. Although a coupler at millimeter wavelength would most likely use some sort of quasi-optical coupler, the basic concept of the coupling can be demonstrated very simply. Imagine a beam propagating through a parallel plate capacitor, connected to an external coupling impedance Z as shown in Fig.(2). The capacitor voltage is V. If the beam current is denoted J, the energy conservation equation for the circuit is

$$d/dt\,[CVV^*/2] + VV^*/Z = -\int d^3r\,\mathbf{E}^*\bullet\mathbf{J} \qquad (18)$$

On the right hand side, E is the field in the capacitor. This field is $(V/d)\,r\cos\theta$, where d is the separation between the plates in the capacitor. Since the electric field has a $\cos\theta$ dependence, it will couple to a p=1 mode on the beam, for either a rotating or filamentary beam. This is what we found in I for the latter case. The calculation of the right hand side of Eq.(18) is quite analogous to the calculation of fluid energy input per unit length, done in the last section, except that now there is also an integral over z for the length of the capacitor.

We find that the equation for the circuit becomes

$$-i\omega CV + V/Z = -(n_o\pi r_b^2 e/d)(\omega/\omega')\int_o^L dz\, v_r(z)\exp ikz \qquad (19)$$

where the velocity is now written as a function of z time the exponential. The velocity of the beam is driven by the fields in the capacitor, or

$$[\exp ikz]dv_r/dz = eV/md \qquad (20)$$

Equations (19) and (20) are two coupled linear equations for $v_r(z)$ and V. Notice that we have not specified up to now whether the beam is filamentary or rotating. The only place this arises is in the expression for ω', the dispersion relation. In either case, $\omega' = \pm|\Omega'|$. We expect the best coupling, combined with cooling of the beam to be with a positive energy wave near k=0. However for the filamentary beam, this is a wave with $\omega\approx\Omega_c$, whereas for a rotating beam is a wave with $\omega\approx 2\Omega_c$. Thus, just by going to a rotating beam, we make the natural coupling at the second harmonic of the cyclotron frequency rather than at the cyclotron frequency.

Substituting from Eq.(19) into Eq.(20), we find

$$(\exp ikyL)dv_r/dy = -A\int_o^1 dy'v_r(y')\exp iky'L \qquad (21)$$

where y=z/L and

$$A = I(L/d)^2(\omega/\omega')/4V_{bz}(-i\omega C+1/Z) \qquad (22)$$

where I is the beam current and V_{bz} is the beam voltage based on the parallel velocity only. Defining T by

$$T = \int_o^1 dy' v_r(y') \exp iky'L, \qquad (23)$$

one can solve Eqs.(21) and (23) and get

$$T = v_r(y=0)\{(\exp ikL - 1)/ikL\}/\{1-A[(ikL)^{-1} + (\exp ikL - 1)/(kL)^2]\} \qquad (24)$$

Now we can insert this back into the equation for v_r and find $v_r(y=1)$. It is

$$v_r(L) = v_r(0)\{1-[4A/(kL)^2]\sin^2(kL/2) /[1- A[(ikL)^{-1} + (\exp ikL - 1)/(kL)^2]\} \qquad (25)$$

At k=0, the result is simply

$$v_r(L) = v_r(0)\{(1-A/2)/(1+A/2)\} \qquad (26)$$

so that $v_r(L)$ vanishes if A=2. This means that the imaginary part of A=0, so that the imaginary part of Z is inductive and is tuned so that the resonant frequency is equal to ω. Then A=2 if $R=8(\omega/\omega')V_{bz}d^2/IL^2$. For a filamentary beam, $\omega=\omega'$, but for a rotating beam, $\omega=(p+1)\omega'$. At this point, the noise input is all dissipated in the external circuit. If there is an external source driving the circuit with A=2, there is maximum coupling to the beam here, exactly as in I. One can also match the slope of the imaginary parts of the beam and circuit impedance as a function of frequency near the resonance, so as to maximize the bandwidth where the noise is minimized. This gives the complex impedance matching condition as in I, and choosing the circuit parameters to this way allows maximum bandwidth of the coupling and noise reduction.

To summarize, a Cuccia coupler works the same way for a rotating beam as it does for a filamentary beam. However the coupling to the filamentary beam is most effective at $\omega=\Omega_c$, whereas the coupling to the rotating beam is most effective at $\omega=2\Omega_c$. In either case it is the uniform field of the capacitor, which has a p=1 structure in cylindrical coordinates, coupling to the positive energy, $k\approx0$, p=1 mode of the beam.

For other values of p, the theory is similar except that the analog to the Cuccia coupler must have the appropriate symmetry. For instance, to generate p=2 modes with $\omega=3\Omega_c$, one would need a Cuccia type coupler but with quadrupole symmetry.

IV. Parametric Amplification

Once the mode is excited on the electron beam, the next issue is amplification. As in I, there are two possibilities, parametric amplification powered by an oscillating quadrupole pump at twice the signal frequency, or a dc quadrupole pump which oscillates spatially with wave number $2\omega/v_o$. In the former (the CWPA), the coupler cools both the signal and idler, which are both positive energy cyclotron waves with wave number near zero. In the latter (the CWESA), the idler is a negative energy cyclotron wave with a wave number significantly different from zero. It is not cooled by the coupler and does not even couple strongly to it. However in the parametric amplification process, noise on the idler does couple back to the signal, so to be a low noise amplifier, the idler noise temperature must also be small. In order to reduce the idler noise, the magnetic field is reduced between the cathode and the interaction region[1]. For millimeter waves, the magnetic field needed in the interaction region is quite high, even at the harmonics, so it is unlikely that a CWESA will be a viable approach, at least for the shorter millimeter waves, although it could be a very interesting approach for the second harmonic at 35GHz.Thus, for now, we concentrate on the CWPA at millimeter waves.

The theory we work out here is very simple and exploits known attributes of parametric amplification. It calculates the power from the pump going into the signal and idler waves, and then uses the fact that the power going into each is proportional to is frequency. If we consider a p=1 mode for both the signal and idler, then the pump should have p=2 in order to satisfy the frequency and wave number matching conditions in time, z and θ. That is the pump should have a quadrupole structure in r and θ. Later, we will discuss briefly other functional dependence on θ.

The pump has functional dependence

$$\phi_p = \phi_o(r/a)^2 \exp -i\omega_p t \pm 2i\theta \; + \; c.c. \qquad (27)$$

Here the $\pm$ on the θ dependence allows for the pump to be circularly polarized in either sense. In the frequency in the exponent, we use the notation that a subscript p means the pump frequency as opposed to the wave number in the θ direction. Now the average energy going into the signal and idler waves is given by

$$-\langle n_i e \mathbf{v_s} \bullet \mathbf{E_p} \rangle + i \leftrightarrow s \qquad (28)$$

The expression for power input is particularly simple because of the assumption of a very tenuous beam which has no electric field. Thus the only electric field comes from the pump. For the signal or the idler, the perturbed electron density is given by Eq.(14), where since we are dealing with only the positive energy cyclotron waves, we take the positive sign for ω'. The θ velocity is related to the radial velocity by Eq.(8). Here, for rotating and filamentary beams, the polarization relations between v_r and v_θ do change sign. This means that the polarization of the pump electric field which excites the instability does change its sense of polarization for parametric instability, depending on

whether the beam is rotating or filamentary. However if the pump is a standing wave in θ, it has equal components of both polarization and will excite the parametric instability for either beam.

Using the expression for the energy density of signal and idler, derived in Section 2, and also the power input form the pump, we find that the spatial growth rate (that is use $v_0 d/dz$ for the time derivative) of the mode amplitude is given by

$$\kappa = e\phi_o/ma^2\Omega_c v_o \tag{29}$$

where we have assumed that only the polarization coupling to the signal and idler is present in the pump, that is, the pump is the proper traveling wave in θ, and not a standing wave. This is virtually the same expression for the spatial growth rate as was the case of a filamentary beam as derived in I. Thus as regards the parametric instability, the rotating beam acts almost as does the filamentary beam, except that the pump is at four times the cyclotron frequency and the p=1 signal is at twice the cyclotron frequency.

Now let us briefly discuss parametric instabilities for signals at other values of p, or at higher cyclotron harmonics. To excite a p=2 mode on the rotating beam, at three times the cyclotron frequency we would use a analog to the Cuccia coupler but with quadrupole field structure in the transverse plane. The parametric instability, of say 2 p=2 modes, each at the third harmonic, would then have to be driven by an oscillating potential with p=4 structure, an oscillating octapole. A derivation analogous to that for the parametric instability of two p=1 modes gives the result for growth rate of two fast cyclotron modes at arbitrary p (assumed positive), denoted p_s and p_i

$$\kappa = \{4e\phi_o(p_s p_i)^{1/2}(r_b/a)^{p_s+p_i-2}\}/\{ma^2(p_s+1)(p_i+1)\Omega_c v_o\} \tag{30}$$

Notice that the as one goes to higher and higher harmonics, the growth rates get smaller, but only in powers of r_b/a. There is no weakening of growth rates in powers of $(v/c)^2$ as there would be in the electromagnetic instabilities. Thus by producing a rotating rather than a filamentary beam, and exploiting electrostatic, rather than electromagnetic effects, one can develop cyclotron wave amplifiers at very high harmonics with very low energy beams.

One very interesting possibility for a 94 GHz CWPA and receiver protector would be to use the third harmonic of the cyclotron frequency and have the idler at the second harmonic. The growth rate for the parametric instability would only be down by a single factor of r_b/a, perhaps a factor of two. However there would have to be two separate Cuccia couplers, a quadrupole coupler at 94 GHz and a dipole coupler at two thirds of this frequency (for cooling only), assuming they could not somehow both be put in the same coupling cavity. However the corresponding advantage is that the magnetic field is only about 12 kG and the pump frequency is only at 155 GHz instead of 190. Also the signal and idler are far apart in frequency and could be easily separated from one another.

V Spinning up the Beam

One of the most difficult aspects of engineering a harmonic CWPA would almost certainly be producing the rotating beam. We will consider several ways of doing this, first of producing a solid beam, and then of producing an annular beam. One way of doing this is to send the beam through a cusp field. The basic nature of a cusp field is that due to the cylindrical symmetry, the canonical momentum in θ direction is conserved. Thus

$$P_\theta = mrv_\theta - erA_\theta/c = \text{constant} \qquad (31)$$

If upstream, $v_\theta = 0$, and $A_\theta = Br/2$, then downstream, in the region of reversed field, that is, if $A_\theta = -Br/2$, the azimuthal velocity is eBr/mc, just the velocity of a rigidly rotating equilibrium. Of course the field must be small enough that the beam can propagate through the cusp. If the cusp is very sharp compared to Larmor diameter, the beam simply becomes a rotating beam. However if the more space the cusp takes, the more likely the beam is to come out with rippling and scalloping motion on the far side of the cusp.

Since the final beam has very small radius (for instance for a second harmonic CWPA at 94 GHz with an electron beam 300 volts rotational energy, the beam radius is only about 40 μm), it may be preferable to produce the beam at lower magnetic field and then compress the field. Since adiabatic compression preserves flux contained in the orbit, it preserves the rigid rotor nature of the equilibrium. Let us consider the cusp at 1 kG, and a beam velocity of 10^9 cm/s, roughly a 300 eV beam. The field must compress from -1 kG to 1 kG in a distance small compared to half a millimeter. Going to somewhat higher voltage, or lower field (but with more subsequent field compression) would relax the length requirement for the cusp. In the interaction region, the beam must be compressed to about 17 kG if the amplifier is to be a second harmonic CWPA 94 GHz. If we consider a beam with a current of 100μA, at the cathode, in the low field region, the emission density is only about 0.1 A/cm^2, not a difficult value to achieve at all. A schematic of the magnetic field profile is shown in Fig.(3).

Since this beam is assumed to be generated at a single energy, once it transits the cusp, and picks up a significant rotational energy, there will be a large energy spread on the axial motion. For a cyclotron wave CWPA this will not affect the performance at the band center, since there is no parallel wave number in this case. However this large spread in v_z will undoubtedly greatly reduce the bandwidth, since away from band center k_z does come into play. However this velocity spread almost certainly renders inoperable the CWESA, which requires k_z for the pump and idler. One possible solution to this would be to use a segmented cathode illustrated schematically in Fig.(4); different portions of the cathode would be a different potentials. The lowest energy electron would be produced at the center, the highest at the edge.

We now discuss the prospect of annular rather than filamentary beams. While the analysis of the Cuccia coupler and parametric instability was done for solid rotating beams, the results for annular rotating beams should not be very different. Annular beams are not only simpler to produce, but they also have much less axial velocity spread, so that a CWESA could be a viable option as well as a CWPA. (Of course if the rotating beam is produced with a segmented cathode, a CWESA could also be viable with a solid beam.) Annular beams are commonly used in gyrotrons, and recently Litton corporation has developed an annular rotating beam for potential operation in a harmonic gyro amplifier[19,20,21]. The Litton gun however uses mostly coils rather than permanent magnets. In some ways, an annular beam is easier to engineer than a solid beam because only the beam produced at one radius has to go through the cusp and produce a non rippling, non scalloping beam on the other side. The condition for propagation through the cusp (like that developed in Eq.(31)) depends only on the magnetic flux enclosed by the annular emitting strip on the cathode. The Litton gun used a permanent magnet inside the cathode to provide the required flux, and has the emitting annulus on the cathode in a nearly field free region. Since the electrons are emitted in a nearly field free region and with no v_θ, they all have the same A_θ at birth, so when the transit the cusp, they will all have the same radius and rotation speed. To aid in transiting the cusp, the Litton gun uses electrostatic focusing.

Another way to get a rotating annular beam is to use rf acceleration of the electrons[22,23]. A filamentary beam is injected down the center of an rf cavity which is excited at the cyclotron frequency. The rf fields accelerate the electrons in the transverse plane and produce a rotating beam. Since the radius of the electron beam as it enters the cavity is small compared to the final Larmor radius, a rotating beam is generated with very little ripple. In practice, this is simple to do and has often been used as source for rotating beams. The great advantage of this scheme as opposed to cusp injection is that it is done in uniform magnetic field, so the magnetic configuration is much easier to engineer. In the millimeter wave CWPA however there is another consideration and that regards current limitation of the beam. As mentioned, the initial radius of the filamentary beam is small compared to the final Larmor radius of the rotating beam. For an electron with 1keV of transverse energy in a 17 kG magnetic field, as required for a second harmonic 94 GHz CWPA, the larmor radius is only about 50 μm. If we assume that the initial radius of the beam is 10μm, and is produced at10 A/cm^2, the maximum current will be about 30 μA. This is not so much less than is normally used in a CWPA, and it may be viable for some applications.

The rf spin up of the beam works well in practice, but involves a separate microwave system, where the CWPA already has one microwave system for the pump radiation. An alternate scheme is to utilize a wiggler the spin up the beam. Instead of a magnetic wiggler, as is common in free electron laser applications, we consider her an electrostatic wiggler. We consider a dipole electrostatic field oscillating in z with wave number $k_w = \Omega_c/v_o$. This could be produced in a very simple way with two binary combs as used in I to generate the quadrupole electrostatic pump for the spatial wiggler in the CWESA. For the quadrupole wiggler, teeth on the two combs are opposite one another so

as to produce quadrupole fields. For the dipole wiggler, opposite teeth on the two combs would be displaced longitudinally from one another by half a wavelength, so as to generate a dipole electrostatic wiggler. A schematic of the quadrupole and dipole double binary comb is illustrated in Fig.(5).

For the wiggler, we assume the functional form

$$\phi_w(x,z) = \phi_w \sinh k_w x \cos k_w z \tag{32}$$

Then in the linear regime, the simplest expression for the perpendicular velocity is

$$v_\perp = e k_w \phi_w L / v_o \tag{33}$$

where L is the length of the electrostatic wiggler system. If this is half a centimeter, and v_o is 2×10^9 cm/s, corresponding to a 1keV beam, we find that $v_\perp$(cm/s) = 2.5×10^5E(V/cm). If a perpendicular velocity of 10^9 is needed, we find that the electric field is about 4000 V/cm. The wavelength of the wiggler is 0.4 mm. The final Larmor radius of the beam is about 30 μm. Thus if the separation of the comb teeth is about 100 μm, the voltage between the two binary combs should be about 40 volts. These fields and voltages are not particularly large or technologically unreasonable. However the manufacture of these combs with such small dimensions may be more difficult. Of course in a careful design, the nonlinear orbits would have to be accurately calculated, but there are standard tools for doing this. However the fact that the production of rotating beams with rf accelerators is so relatively simple, and fact that the resonant dipole electric wiggler exploits the same physical process, leads us to conclude that this is probably the simplest and most effective approach to generating a rotating annular beam for the CWPA. Furthermore, while this might produce a rotating beam with rippling and scalloping motion, this motion at $2\Omega_c$ would be effectively removed by the Cuccia coupler.

While an annular rotating beam is almost certainly simpler to generate than a solid one, the annular beam is subject to instabilities, while the solid beam is not. The classic instability of annular beams, either filamentary or rotating is the diocotron instability[24]. Its growth rate is a rough measure of the growth rate for all modes. The spatial growth rate of the diocotron instability is given by $\omega_p^2/2\Omega_c v_o$ where ω_p is the plasma frequency of the beam. For an annular rotating beam to be viable, especially for a low noise amplifier, the growth rate of the diocotron instability should be considerably less than the growth rate of the parametric instability in the amplifying region. This is ultimately a limitation on the beam current, since the diocotron instability depends on the self fields of the beam, whereas nothing else worked out here does. Comparing the growth rates of the parametric and diocotron instability, we find the latter is very small if

$$m a^2 \omega_p^2 / 2 e \phi_p << 1 \tag{34}$$

If we consider a 30 μA beam with a radius of 40 μm and a thickness of one tenth of this, Eq.(34) is easily satisfied for the sorts of dimensions and pump potentials we have been considering here. Thus it is probably true that a harmonic CWPA with a rotating annular beam is the simplest way to attempt an initial experiment.

Let us finally discuss briefly the possibility of a two beam CWPA with a rotating beam. As we recall from I, in a CWPA, one disadvantage is that the signal and idler are very close in frequency. Thus if the signal is downshifted in frequency with a mixer, and the downshifted signal is passed through a low pass filter, if the signal and idler are sufficiently close in frequency, they will both pass through the filter. Hence there is a small region in the center of the band where the CWPA is not usable. One solution, discussed in I, was the use of a two beam CWPA. There are two separate beams, and on the two beams, the two pump signals are 180° out of phase. The two beams amplify the signal and idler, however the amplified idler signals are 180° out of phase with one another, so when the signals from the two beams are coherently combined, the two idler signals destructively interfere with one another and essentially vanish.

For a rotating beam CWPA, the same thing is possible. However if the beam is formed by a cusp, which is inherently cylindrically symmetric, it seems that it would be very difficult to generate two rotating beams. For the rotating beam generated by the electrostatic wiggler, it seems that it would be a much simpler matter. There would just be two separate electrostatic wigglers in the beam transport system.

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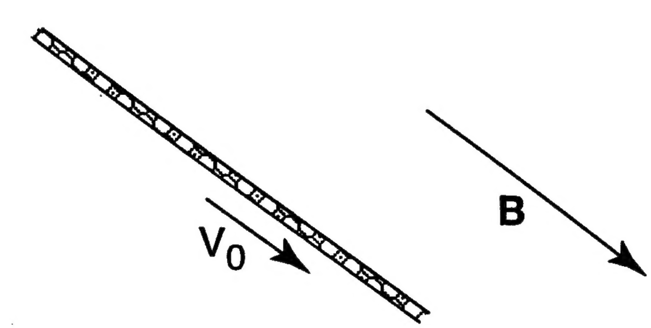

Fig. 1(a) - A filamentary beam

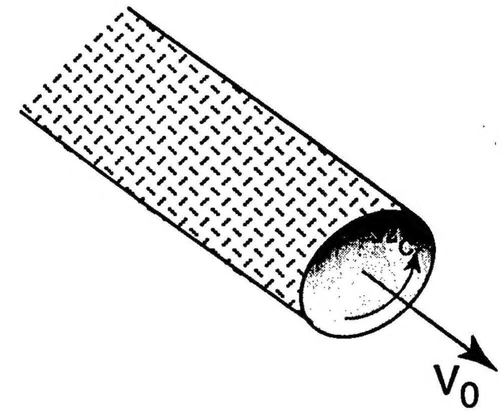

Fig. 1(b) - A rotating beam

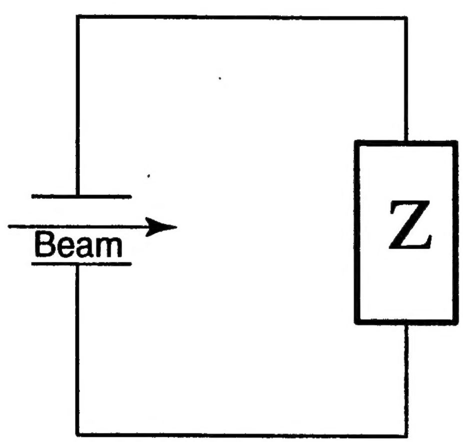

Fig. 2 - The simplified circuit of a Cuccia coupler.

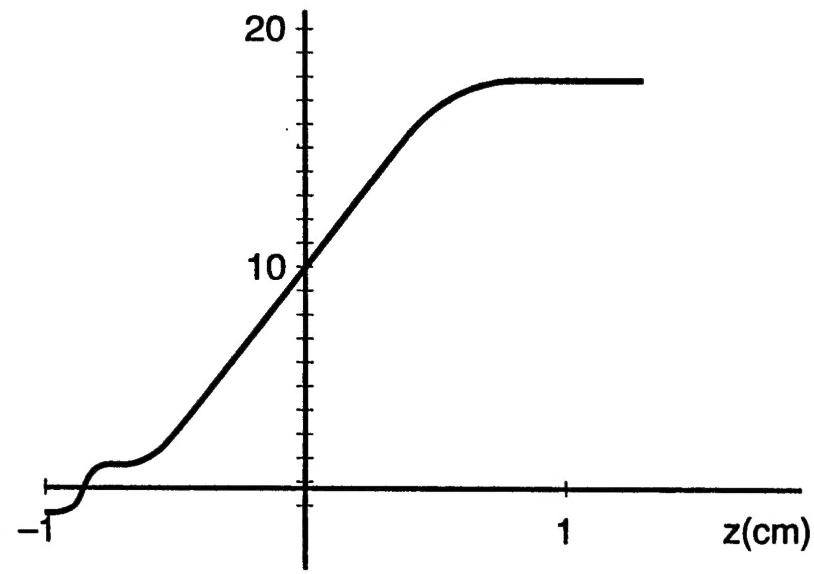

Fig. 3 - The axial structure of the cusp field.

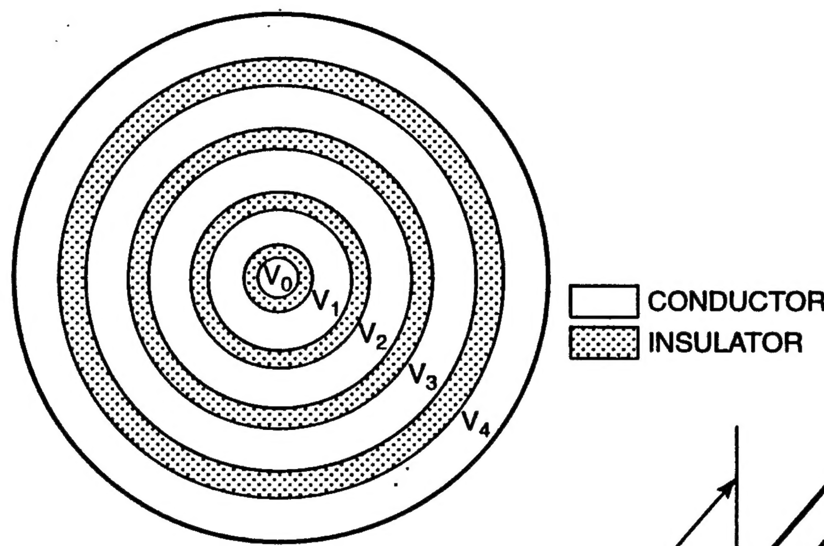

Fig. 4 - The segmented cathode.

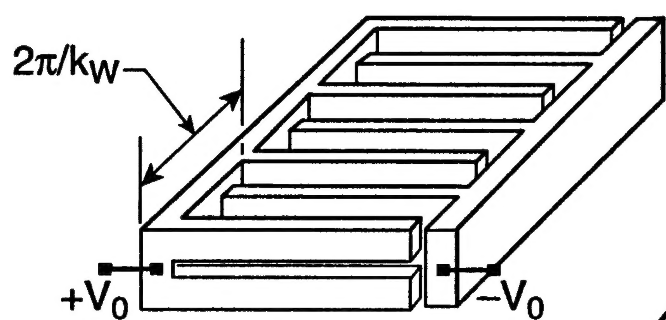

Fig. 5a - The double binary comb in a quadruple configuration.

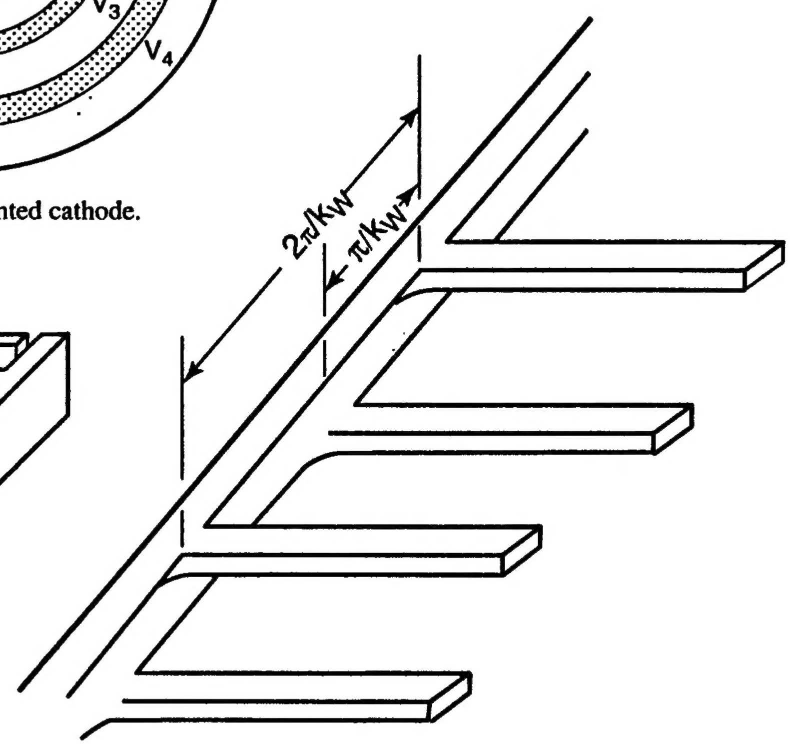

Fig. 5b - One of the two binary combs for the dipole configuration.